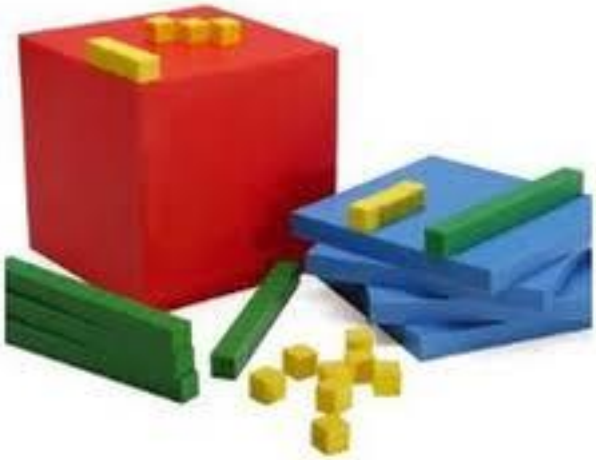
CaPturing mAths

CONCRETE PICTORIAL ABSTRACT

2 + 1 = 3

Calculation Policy



This policy has been designed to teach children through the use of concrete, pictorial and abstract methods. This calculation policy should be used to support children to develop a deep understanding of number and calculation.

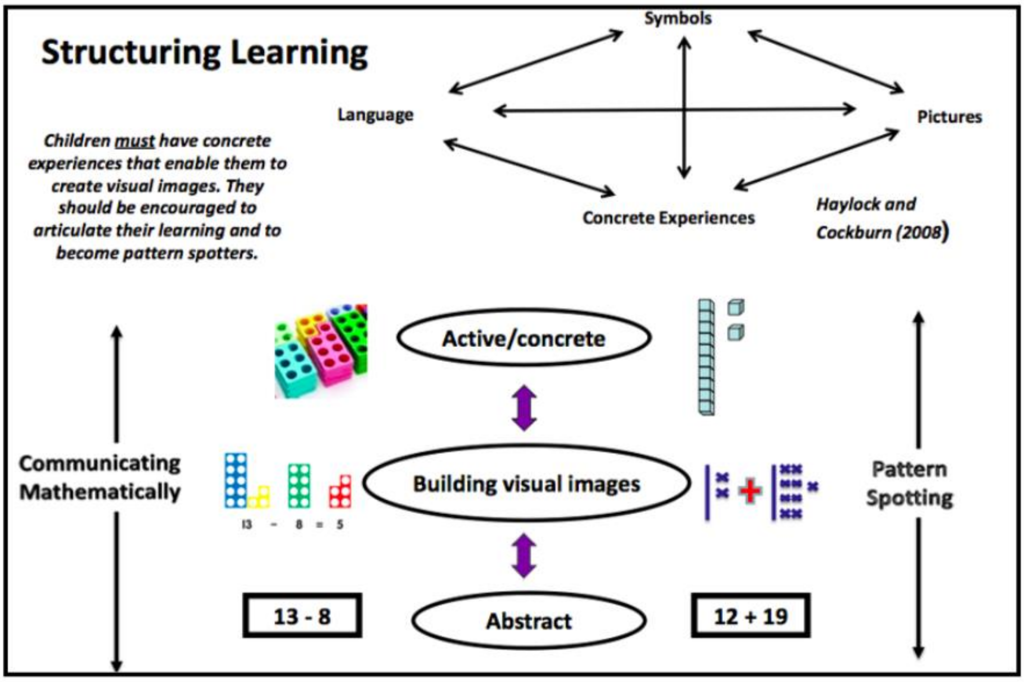
Background
This policy has been developed by Maths Coordinators with a specific interest in the use of Singapore methods to develop number awareness and fluency.
The policy only details the strategies; teachers must plan opportunities for pupils to apply these; for example, when solving problems, or where opportunities emerge elsewhere in the curriculum.

Using the concrete-pictorial-abstract approach:
Children develop an understanding of a mathematical concept through the three steps (or representation) of concrete-pictorial-abstract approach. Reinforcement is achieved by going back and forth between these representations.

Concrete representation The enactive stage - a pupil is first introduced to an idea or a skill by acting it out with real objects. This is a 'hands on' component using real objects and it is the foundation for conceptual understanding.

Pictorial representation The iconic stage - a pupil has sufficiently understood the hands-on experiences performed and can now relate them to representations, such as a diagram or picture of the problem.

Abstract representation The symbolic stage - a pupil is now capable of representing problems by using mathematical notation, for example: 12 ÷ 2 = 6 .

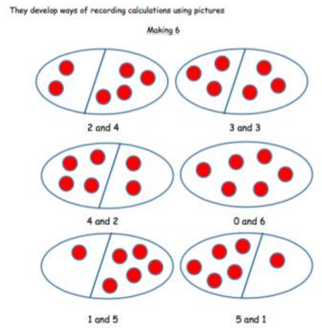


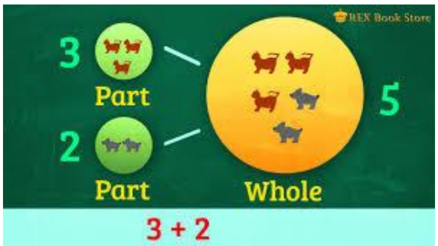
Guidance
This is document provides guidance and examples for key objectives for each year group but is not to be followed as a complete planning aid as not all objectives are exemplified.

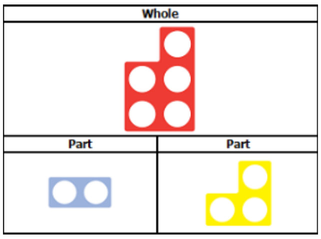
Reception

Addition

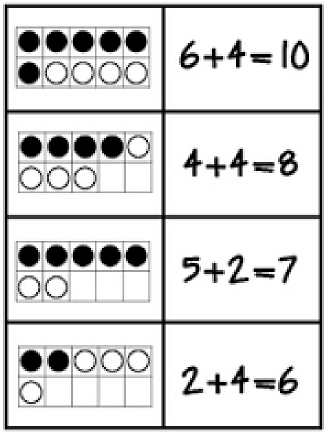
Explore part part whole relationship



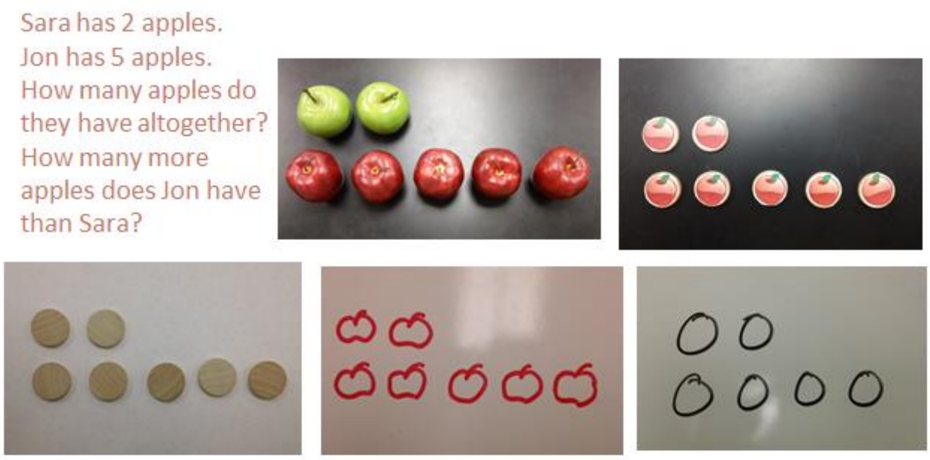




Using the ten frame to support addition of single digits – counting all/combining two groups

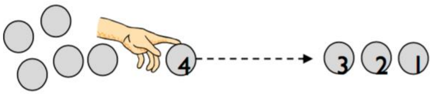


Solving problems using concrete and pictorial images.



Subtraction

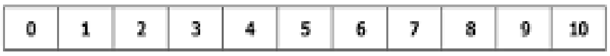
Taking away after counting out practical equipment. . Children would be encouraged to physically remove these using touch counting.



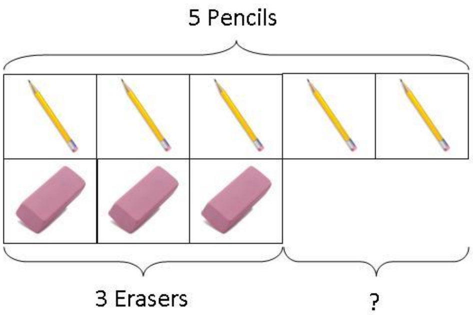
By touch counting and dragging in this way, it allows children to keep track of how many they are removing so they don't have to keep recounting. They will then touch count the amount that are left to find the answer.

Those who are ready may record their own calculations

$8 - 4 = \_\_$



Using the ten frame to support subtraction by taking away



- Peter has 5 pencils and 3 erasers. How many more pencils than erasers does he have?

Solving problems using concrete and pictorial images.

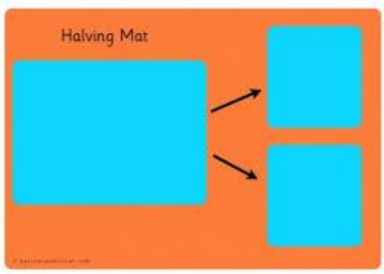
Multiplication

Children will experience equal groups of objects.

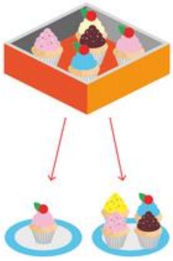
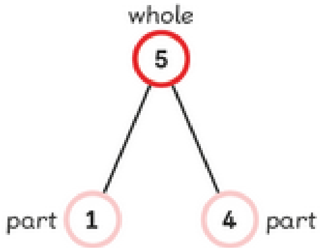
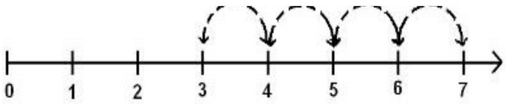
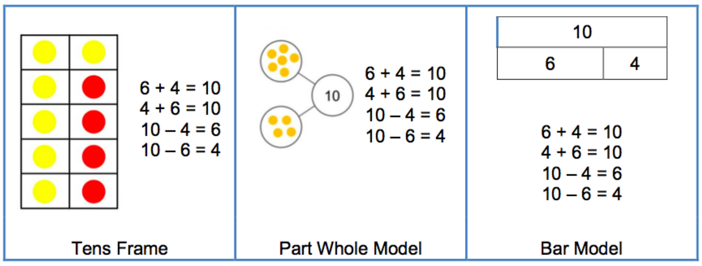
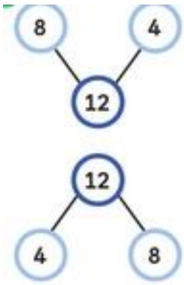
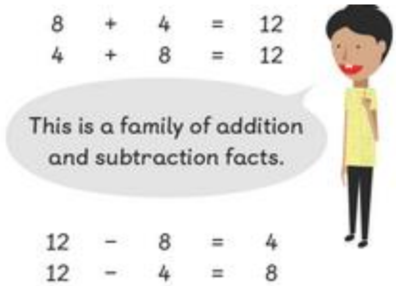
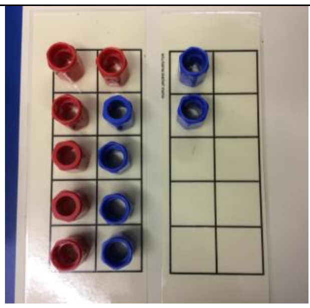
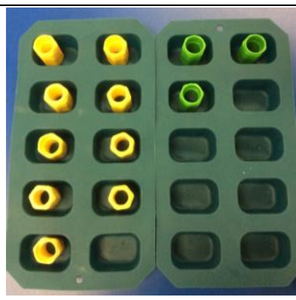
They will work on practical problem solving activities involving

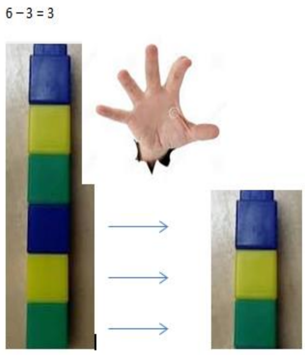
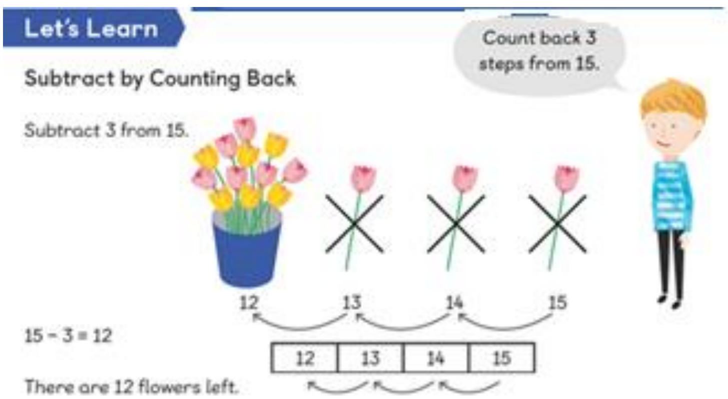
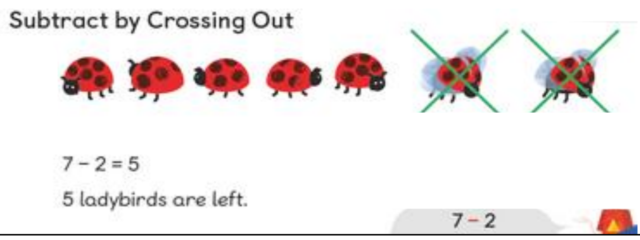
There are 6 pairs of socks. How many socks are there altogether?

Division



Year 1

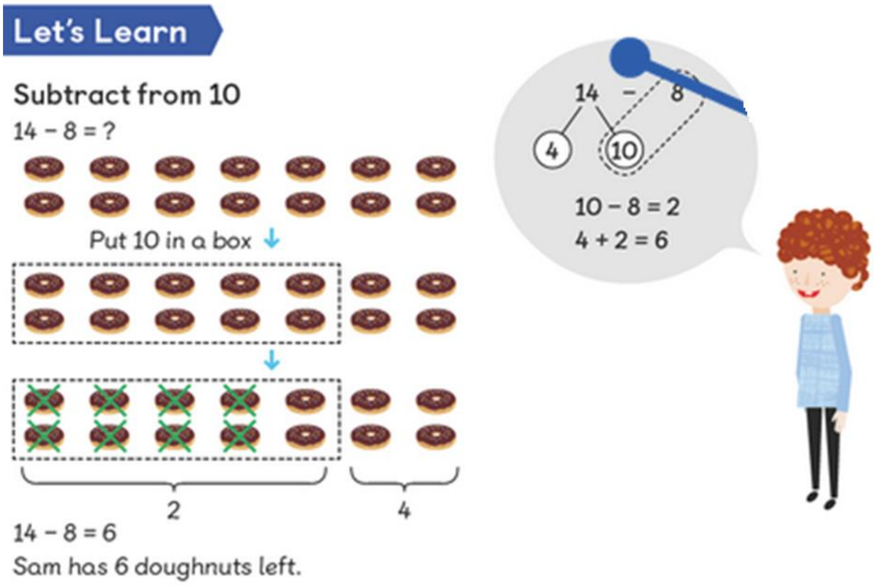
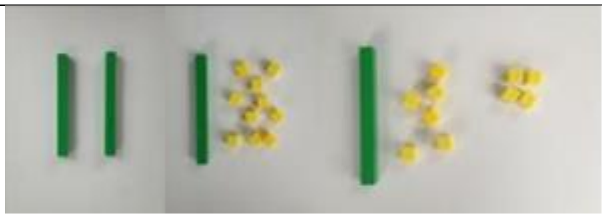
Addition	
Joining two groups and then recounting all objects using one-to-one Correspondence (lots of practice making 10 and numbers to 10 e.g. 6 + 4 = 10 or 3 + 5 = 8)	3 + 4 = 7 
Learn number bonds to 20 and demonstrate related facts *Teach addition and subtraction alongside each other as pupils need to see the relationship between the facts.*	
Add and subtract one digit numbers and two digit numbers to 20, including zero	8+1=9 
Bridging 10 Use ten frames, Singapore bars, egg boxes and number lines to practice. *Chn should start with the larger number and add the smaller number seeing what makes ten and what is left over.*	 6 + 6 = 12 Make 9 in one and 3 in the other. Take one from the 3 to make the 9 into a ten….10+2 = 12

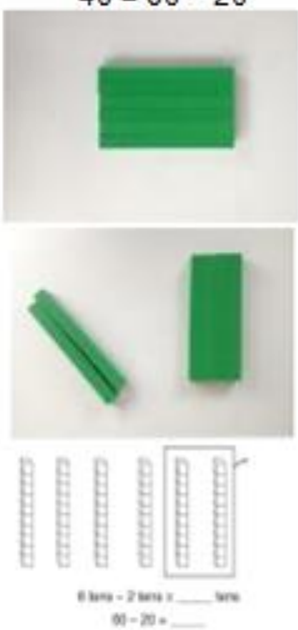
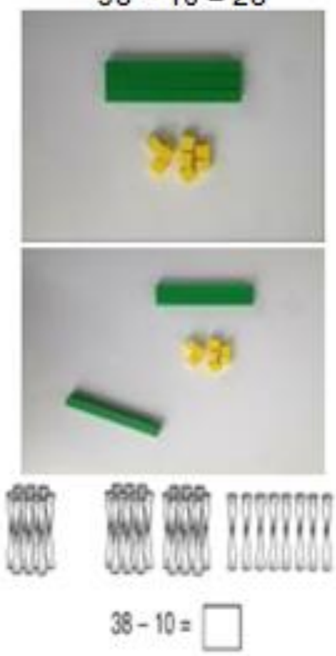
<table>
<tr><th colspan="2">Subtraction</th></tr>
<tr><td>Taking away should begin with physical objects: objects, cubes, Dienes etc</td><td>
</td></tr>
<tr><td>Subtraction by counting back</td><td>
</td></tr>
<tr><td>Subtracting a single digit number from a single digit number and a single digit from a two digit by crossing out pictures</td><td>
</td></tr>
<tr><td>Subtracting using the part part whole (include problem solving with missing digits).

? - 5 = 2</td><td>
</td></tr>
</table>

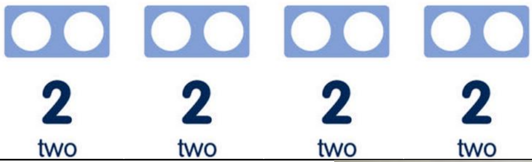
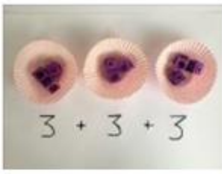
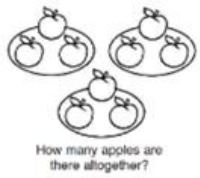
<table>
<tr>
<td>Subtraction by subtracting from 10

Children subtract from 10 and not from ones</td>
<td>14 – 8 = ?

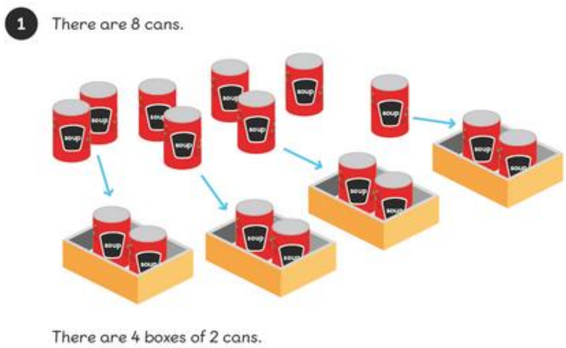

</td>
</tr>
<tr>
<td>When subtracting using Dienes children should be taught to regroup a ten rod for 10 ones and then subtract from those ones</td>
<td>
20 – 4 = 16</td>
</tr>
<tr>
<td>Subtracting multiples of 10

Using the vocabulary of 1 ten, 2 tens etc alongside 10, 20, 30 Is very important here as pupils need to understand that it is a 10 not a 1 that is being taken away</td>
<td>40 = 60 – 20

38 – 10 = 28

</td>
</tr>
</table>

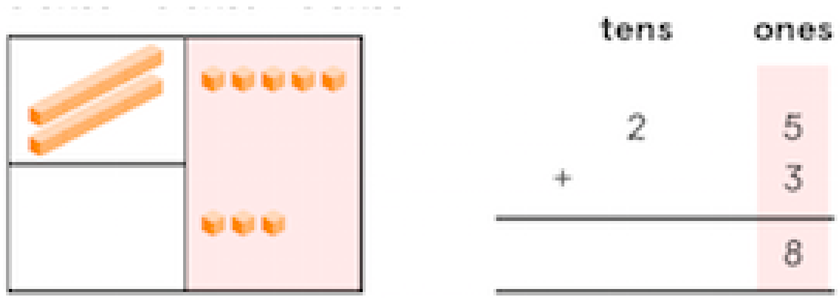
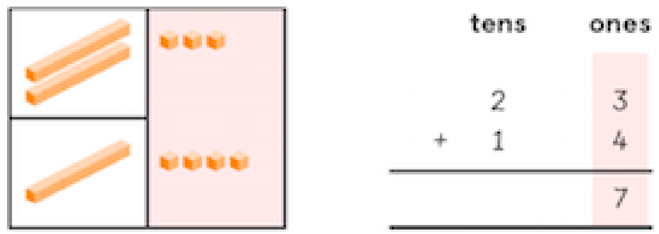
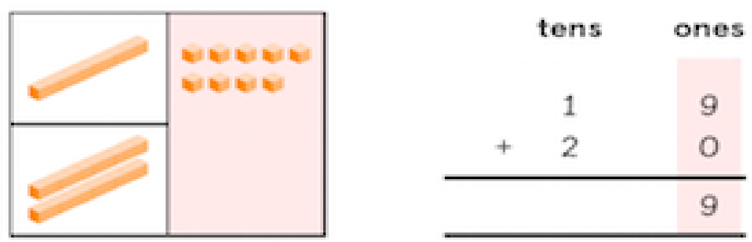
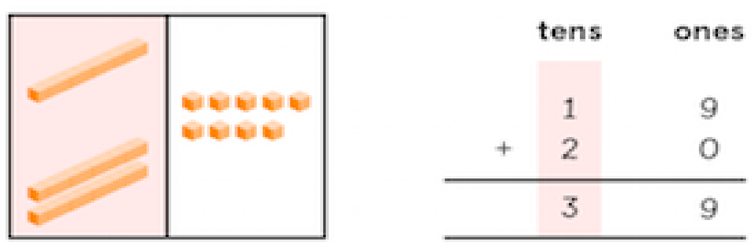
Multiplication

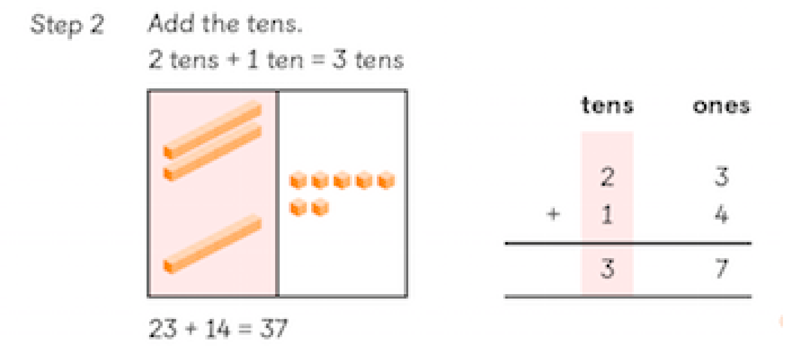
Counting in multiples of 2, 5 and 10 from zero *Children should count the number of groups on their fingers as they are skip counting.*	2 4 6 8 4 groups of 2 = 8  2 × 4 = 8 
When moving to pictorial/written calculations the vocabulary is important	 This image represents two groups of 4 or 4 twice
Solving multiplication problems using repeated addition	  3 + 3 + 3 = 9

Division

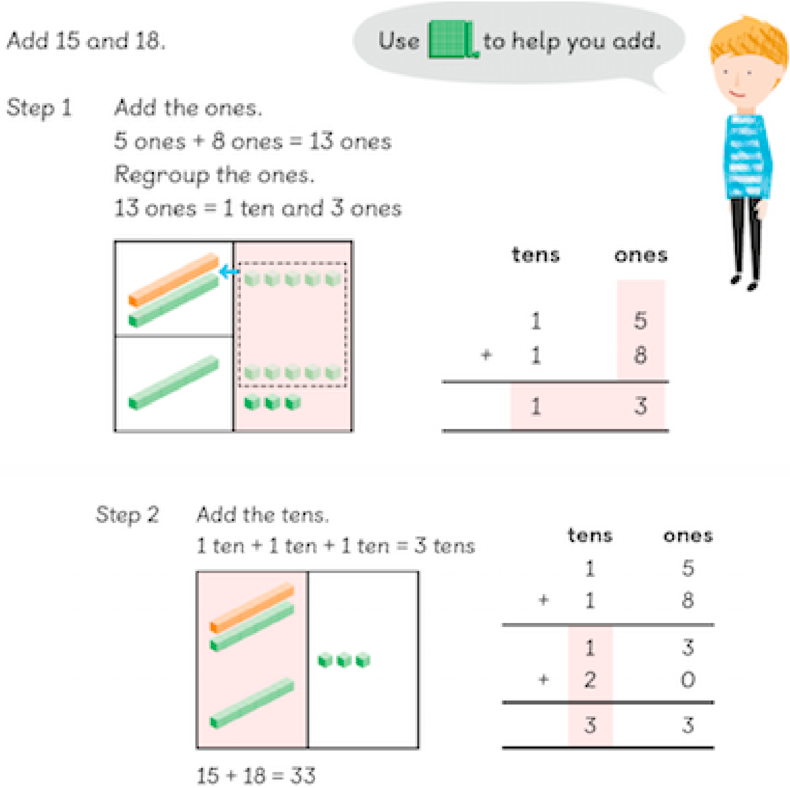
Pupils should be taught to divide through working practically and the sharing should be shown below the whole to familiarize children with the concept of the whole. *The language of whole and part part should be used.*	10 ÷ 2 = 5  8 ÷ 4 = 2

Year 2

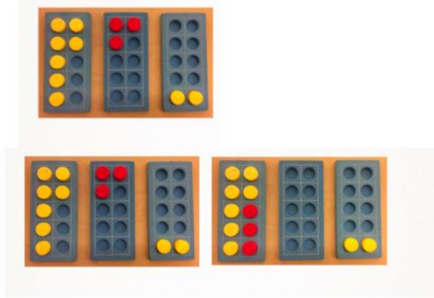
Addition	
Using concrete objects and pictorial representations to add a 2 digit number with a 1 digit number.	
Using concrete objects and pictorial representations to add a 2 digit number and 10s number.	Step 1 Add the ones. 3 ones + 4 ones = 7 ones  Step 1 Add the ones.  Step 2 Add the tens. 1 ten + 2 tens = 3 tens  19 + 20 = 39

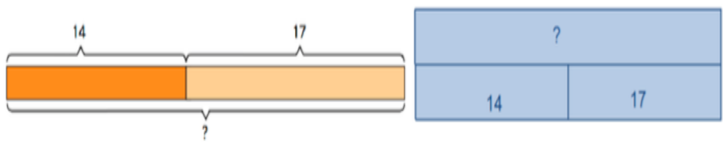
<table>
<tr>
<td>Using concrete objects and pictorial representations to add a 2 2 digit numbers.</td>
<td>


Adding with renaming

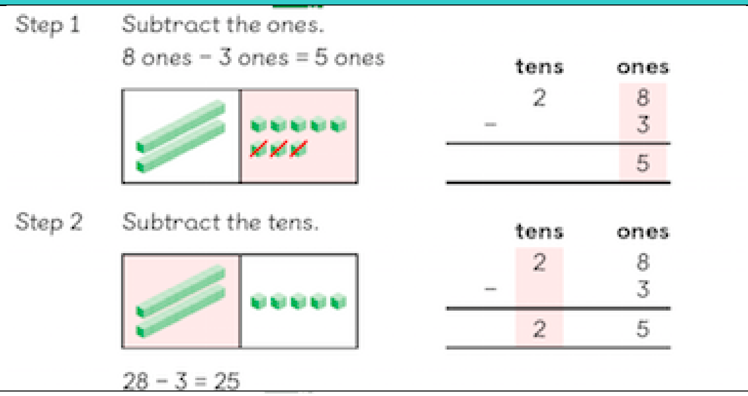
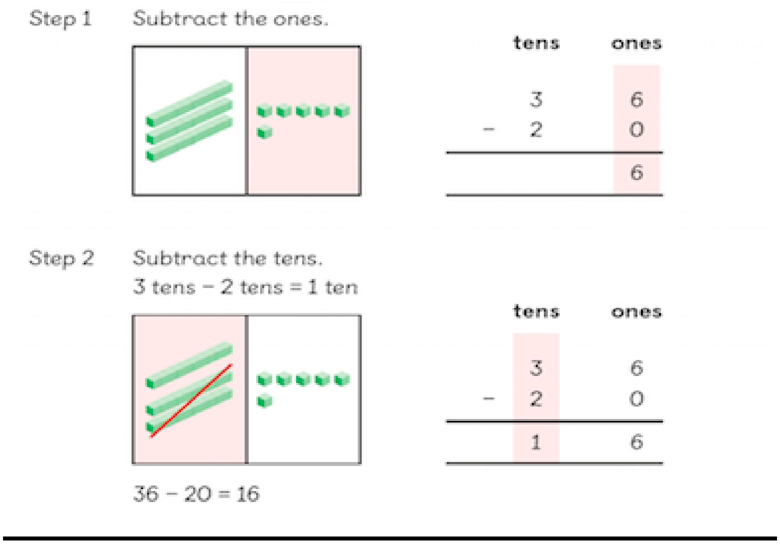
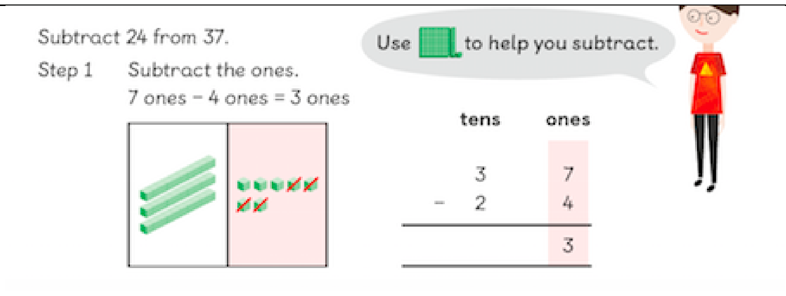


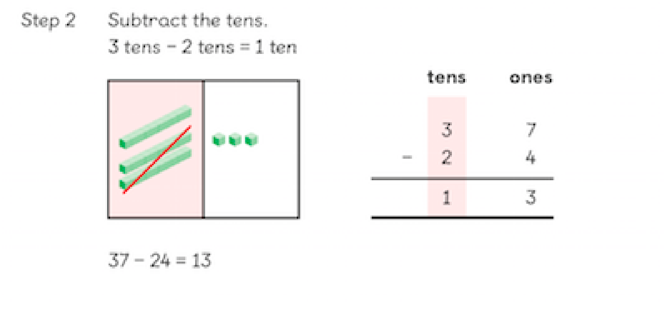
</td>
</tr>
<tr>
<td>Using concrete objects and pictorial representations to add a 3 single digit numbers.</td>
<td>
7+3+2 = leads to 10 + 2 =


</td>
</tr>
<tr>
<td>Using the bar to find missing digits.
It is important for children to use the bar in this way to encourage the use of it to aid with problem solving.</td>
<td>
Helen has 14 bread-sticks. Her friend has 17. How many do they have altogether?

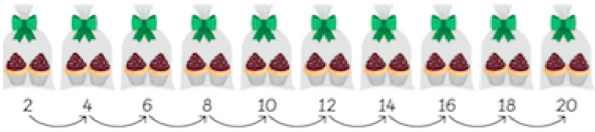
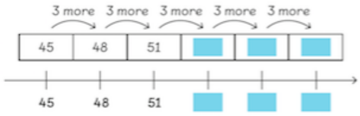
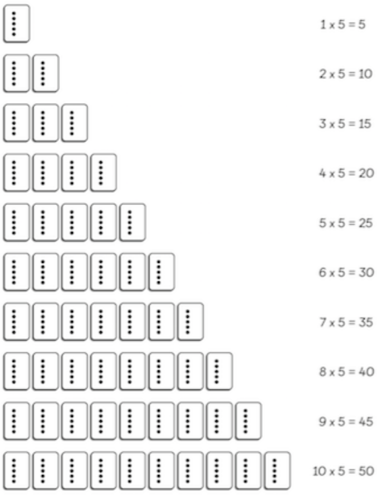
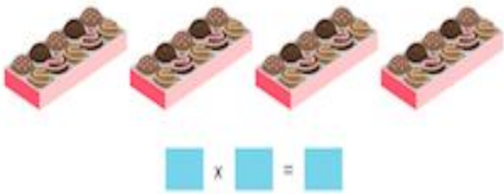
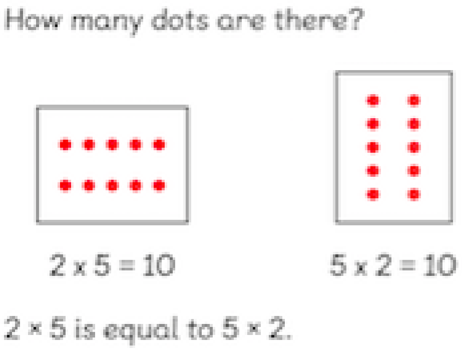
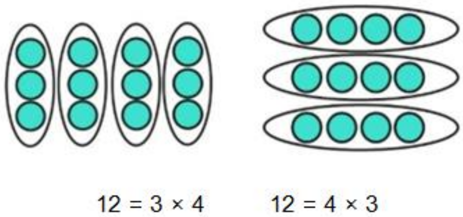
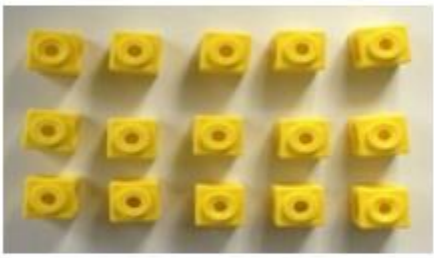
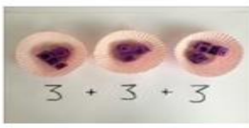
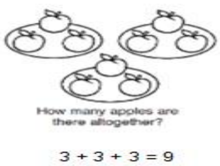


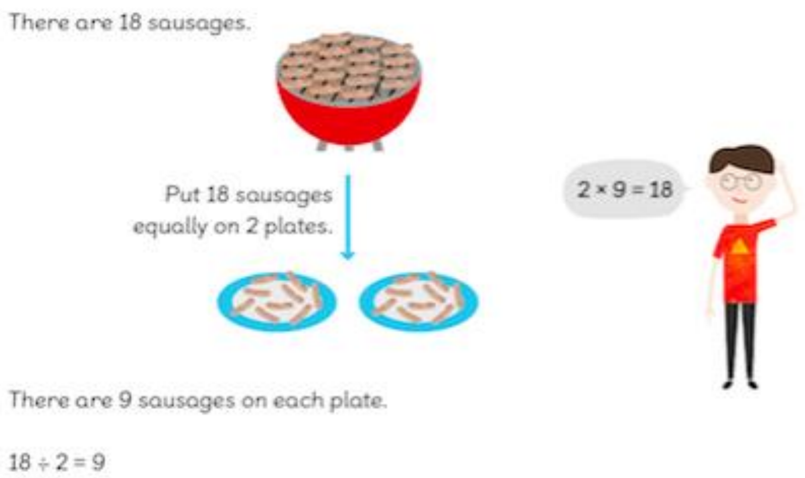
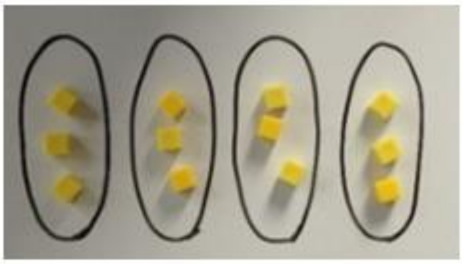
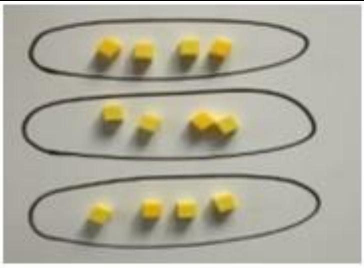
</td>
</tr>
</table>

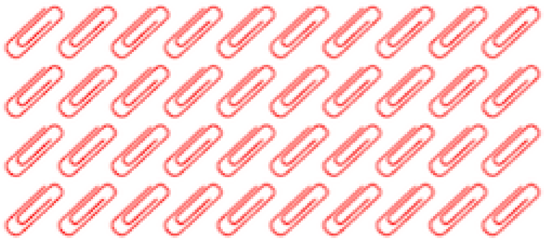
<table>
<tr><th colspan="2">Subtraction</th></tr>
<tr><td>Using concrete objects and pictorial representations to subtract a 1 digit number from 2 digit number.</td><td>
</td></tr>
<tr><td>Using concrete objects and pictorial representations to subtract a 10s number from 2 digit number.</td><td>
</td></tr>
<tr><td>Using concrete objects and pictorial representations to subtract a 2 digit number from 2 digit number.</td><td>

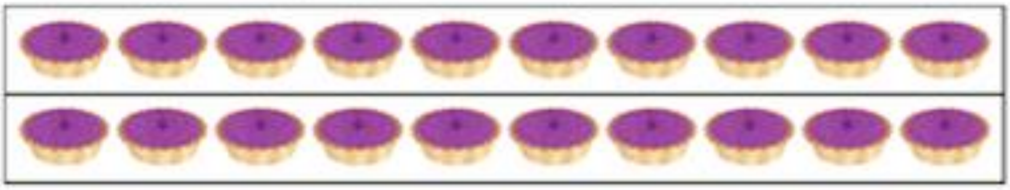

</td></tr>
<tr><td>Recognise and use the inverse relationship between addition and subtraction</td><td><table><tr><td colspan="2">?</td></tr><tr><td>23</td><td>53</td></tr></table>
<table><tr><td colspan="2">76</td></tr><tr><td>23</td><td>?</td></tr></table>
Use this to check calculations and solve missing number problems.</td></tr>
</table>

Multiplication

Skip counting in multiples of 2, 3, 5, 10 from 0	 
Recall and use multiplication facts for the multiplication tables 2, 5 and 10.	
I can use multiplication (x) and equal (=) sign when writing out my times tables.	
Multiplication is commutative *Pupils should understand that an array can represent different equations and that, as multiplication is commutative, the order of the multiplication does not affect the answer.*	 
Solve multiplication problems in context using arrays and repeated addition	 3 x 5 = ☐ 5 x 3 = ☐  

<table>
<tr><th colspan="2">Division</th></tr>
<tr><td>Recall and use division facts for the multiplication tables 2, 5 and 10.</td><td>10 ÷ 10 • • 5
20 ÷ 10 • • 7
70 ÷ 10 • • 2
50 ÷ 10 • • 6
60 ÷ 10 • • 1
100 ÷ 10 • • 10</td></tr>
<tr><td>Solve division problems in context using concrete objects by sharing</td><td>
</td></tr>
<tr><td>Solve division problems in context using arrays</td><td> </td></tr>
<tr><td>I can solve division as grouping.</td><td>Put 10 buns in groups of 2.
How many plates are there?
</td></tr>
</table>

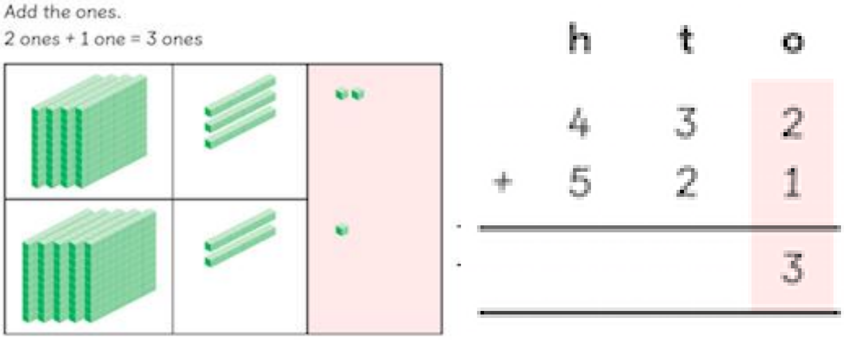
<table>
<tr><td></td><td>
Put into groups of 5.
There are ☐ groups.</td></tr>
<tr><td>I can use the inverse.

This should be taught alongside both multiplication and division.</td><td>Make a family of multiplication and division facts.

2 x 10 = 20 ———— 20 ÷ 10 = ☐
10 x 2 = 20 ———— 20 ÷ 2 = ☐</td></tr>
</table>

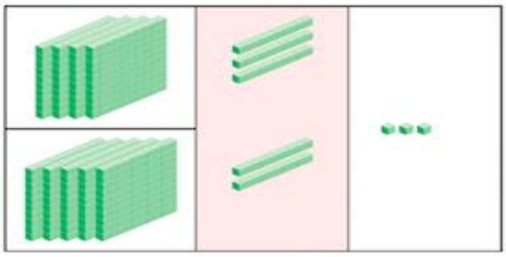
Year 3

<table>
<tr><th colspan="2">Addition</th></tr>
<tr><td>Add two three digit numbers.

Children need to use equipment first to support their understanding of place value.

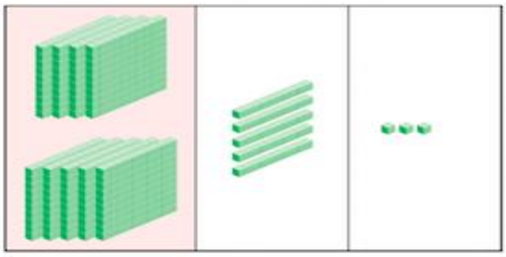
Children to word gradually to three digit + three digit starting without carrying and gradually moving towards carrying.</td><td>432 + 521 =
Step 1 Add the ones.
2 ones + 1 one = 3 ones

</td></tr>
</table>

Step 2 Add the tens.
3 tens + 2 tens = 5 tens



	h	t	o
	4	3	2
+	5	2	1
		5	3

Step 3 Add the hundreds.
4 hundreds + 5 hundreds = 9 hundreds

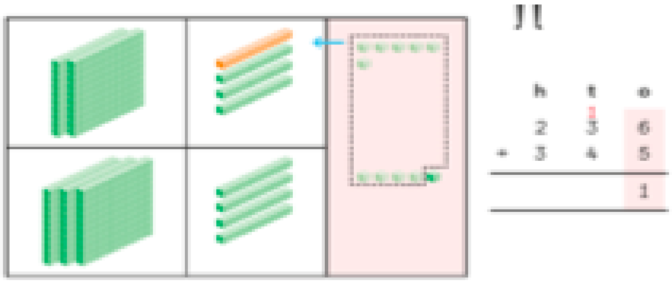


	h	t	o
	4	3	2
+	5	2	1
	9	5	3

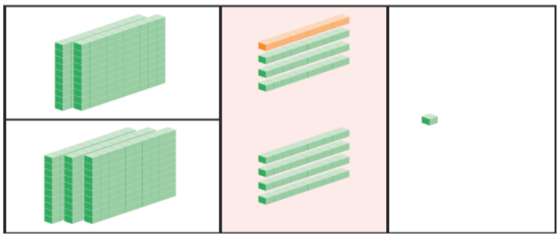
432 + 521 = 953

There are 953 flowers altogether.

236 + 345 =

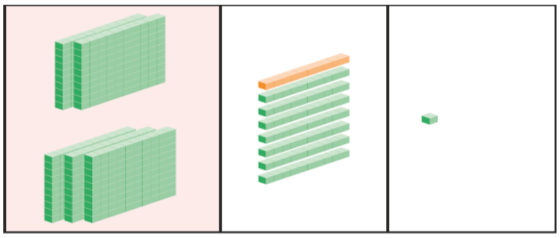


Step 2 Add the tens.
1 ten + 3 tens + 4 tens = 8 tens



	h	t	o
	2	$\overset{1}{3}$	6
+	3	4	5
		8	1

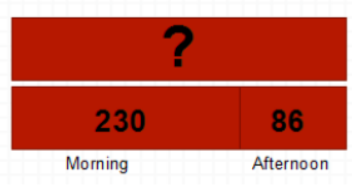
Step 3 Add the hundreds.
2 hundreds + 3 hundreds = 5 hundreds

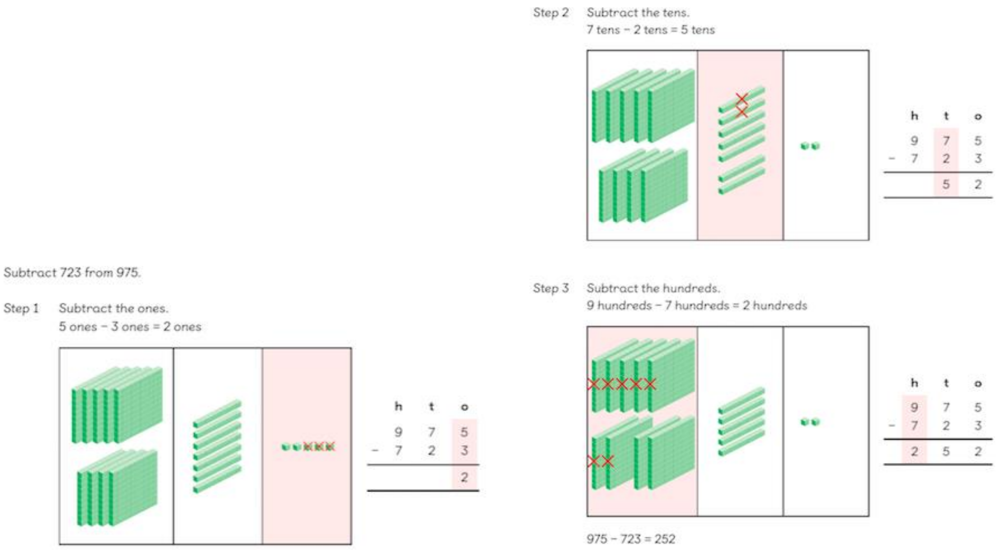


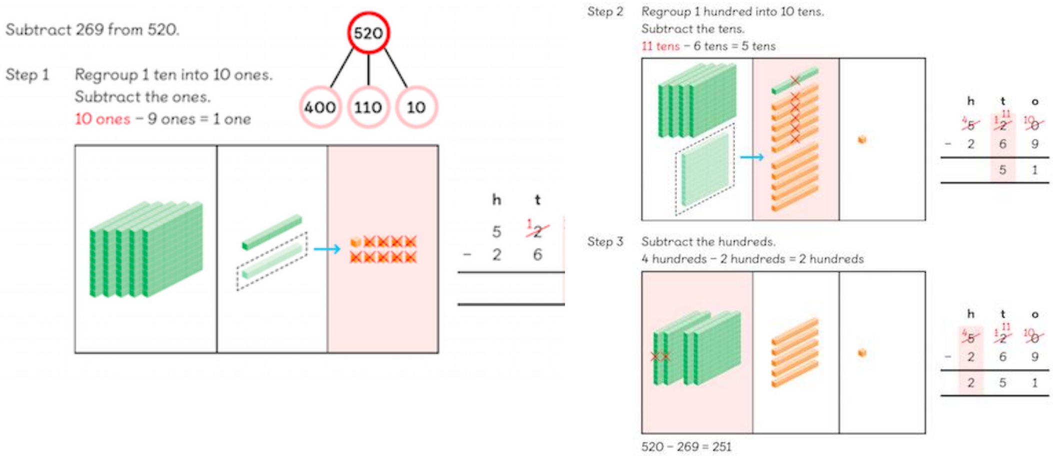
	h	t	o
	2	$\overset{1}{3}$	6
+	3	4	5
	5	8	1

236 + 345 = 581

<table>
<tr>
<td>Using the bar to find missing digits.
It is important for children to use the bar in this way to encourage the use of it to aid with problem solving.</td>
<td><u>Bar Model to support understanding of problem solving:</u>

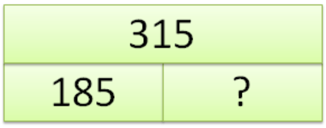
A man sold 230 balloons at a carnival in the morning. He sold another 86 balloons in the evening . How many balloons did he sell in all?

</td>
</tr>
<tr>
<td colspan="2">Subtraction</td>
</tr>
<tr>
<td>Subtract up to 3 digits from 3 digits.

Very important for children to use dienes equipment along with a place value chart to support.</td>
<td>Only when secure with the method should exchanging be introduced.


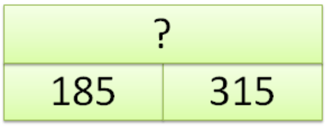

</td>
</tr>
</table>

Using the bar to find missing digits.

It is important for children to use the bar in this way to encourage the use of it to aid with problem solving.



315 – 185 = ?
185 + ? = 315

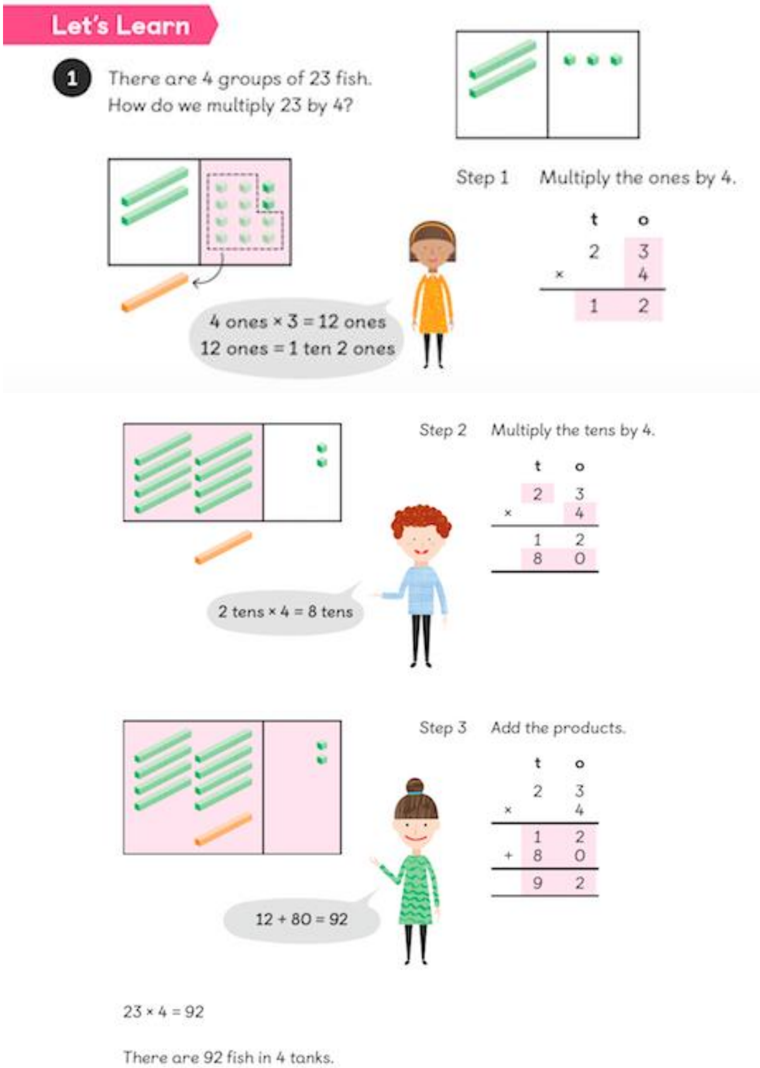


185 + 315 = ?
? – 185 = 315

Multiplication

Children should be able to recall the 2, 5, 10, 3, 4 and 8 times tables.

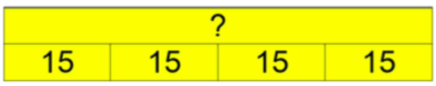
Multiple a two digit number by a one digit.

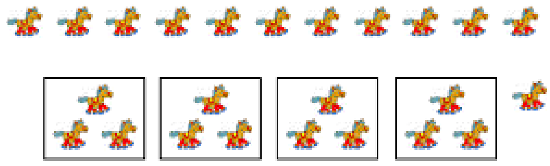


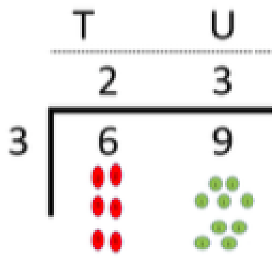
Using the bar to solve multiplication problems.

4 children go to the cinema. They each pay £15. How much do they spend altogether?

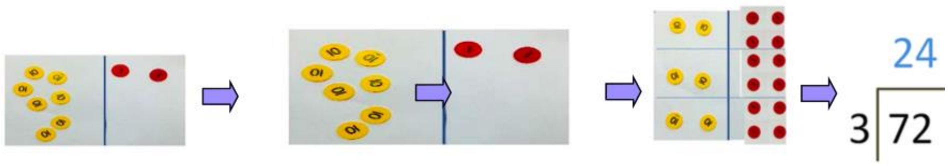
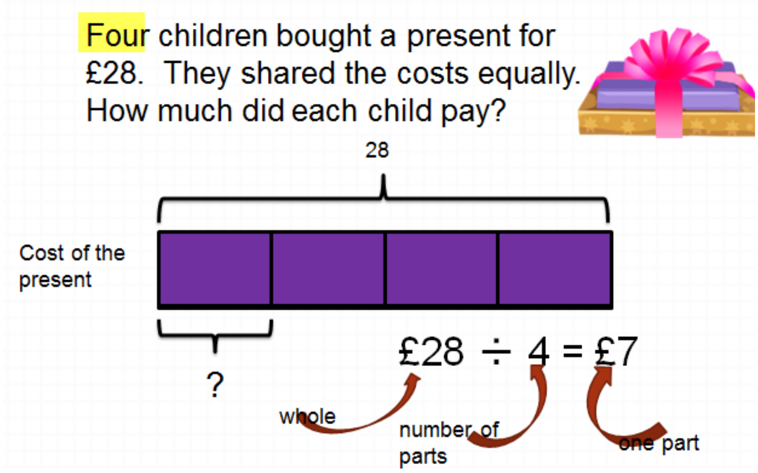
Whole unknown



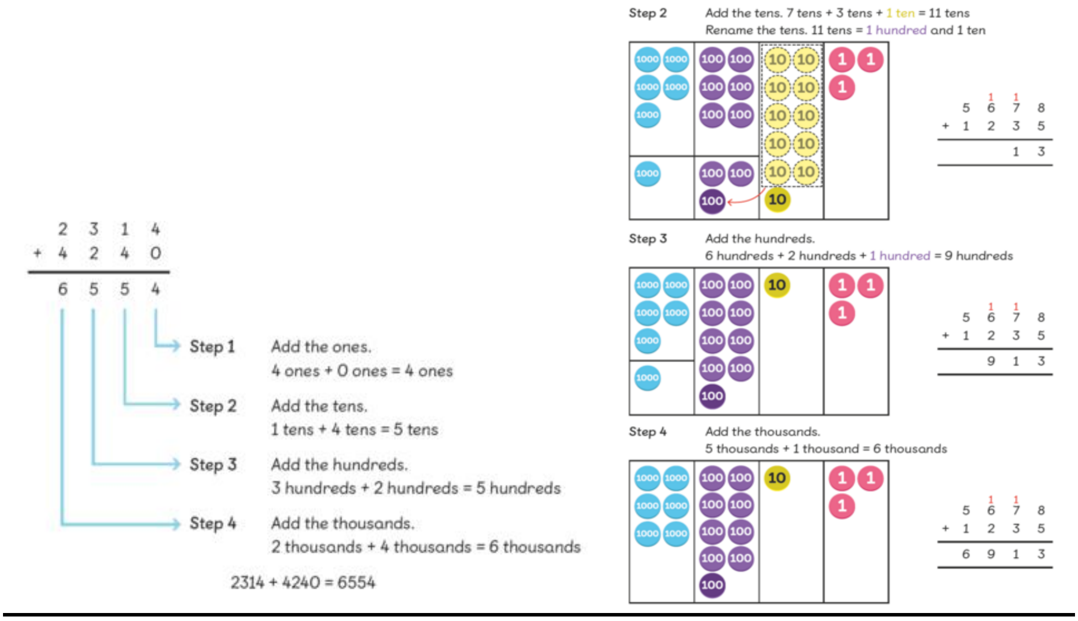
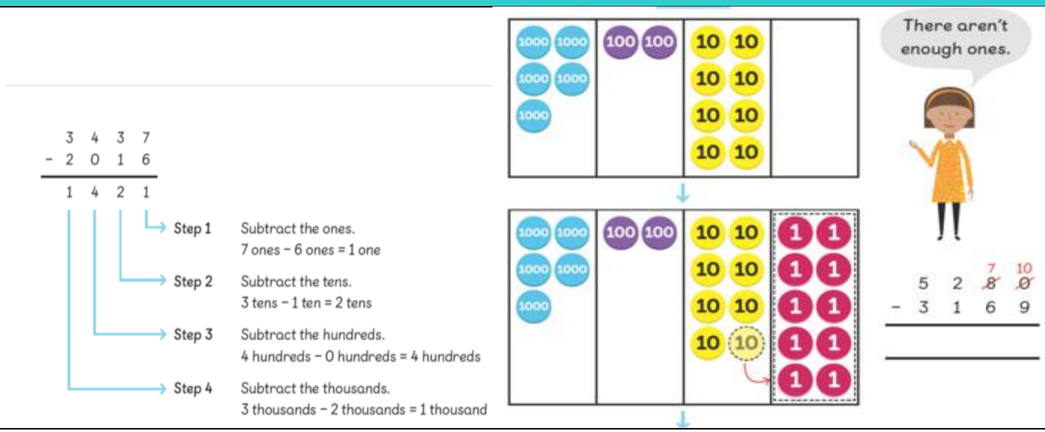
<table>
<tr><th colspan="2">Division</th></tr>
<tr><td>Dividing by grouping undrestanding the concept of remainders.</td><td>Start with using the real objects-or objects that represent the calculation.

13 ÷ 4 = 3 Remainder 1</td></tr>
<tr><td>Dividing using short division.

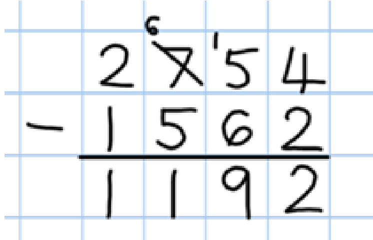
Once children are secure with division as grouping and demonstrate this using number lines, arrays etc., short division for larger 2-digit numbers should be introduced, initially with carefully selected examples requiring no calculating of remainders at all. Start by introducing the layout of short division by comparing it to an array.</td><td>

Remind children of correct place value, that 69 is equal to 60 and 9, but in short division, pose:
· How many 3’s in 6? = 2, and record it above the 6 tens.
· How many 3’s in 9? = 3, and record it above the 9 ones.

Once children demonstrate a full understanding of remainders, and also the short division method taught, they can be taught how to use the method when remainders occur within the calculation (e.g. 72÷3), and be taught to ‘carry’ the remainder onto the next digit.

</td></tr>
<tr><td>Using the bar to aid the solving of division problems.</td><td>
</td></tr>
</table>

Year 4

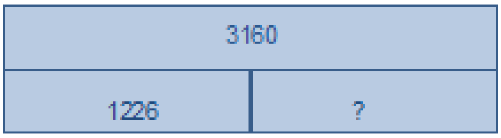
Addition	
Adding numbers with up to 4 digits. *Again this should start with the children using dienes to support them with lots of discussion about the value of each digit.*	
Using the bar to find missing digits. *It is important for children to use the bar in this way to encourage the use of it to aid with problem solving.*	This is not a form of getting the correct answer but helping to guide children to the correct operation. Alison jogs 6,860 metres and Calvin jogs 5,470 metres. How far do they jog altogether? 
Subtraction	
To subtract with numbers up to four digits including exchanging when children are secure. *Again children need to use dienes to support their learning.*	



Using the bar to find missing digits.

It is important for children to use the bar in this way to encourage the use of it to aid with problem solving.

There are 3,160 books in a shop. 1,226 are in English and the rest are in French. How many French books are there?

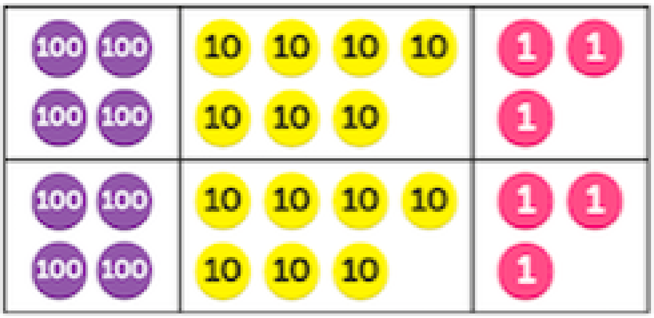


Multiplication

Children to know all times tables to 12 x 12.

Ladder method to be used with children multiplying both two and three digits by a one digit number.

$$\begin{array}{rl} 3\,1\,4 & \\ \times\ 3 & \\ \hline 1\,2 & (3 \times 4) \\ 3\,0 & (3 \times 10) \\ +\ 9\,0\,0 & (3 \times 300) \\ \hline 9\,4\,2 & \end{array}$$

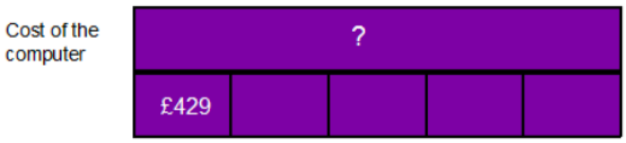


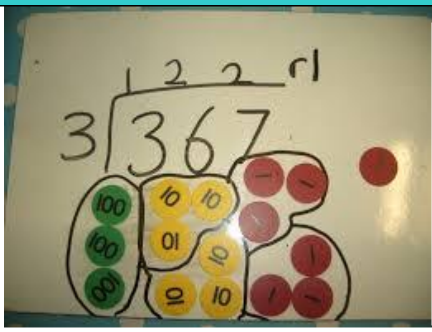
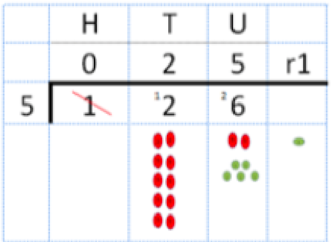
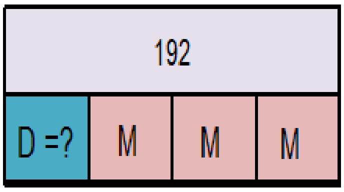
$$\begin{array}{rrr} 4 & 7 & 3 \\ \times & & 2 \\ \hline \\ \hline \end{array}$$

Multiplying using the bar.

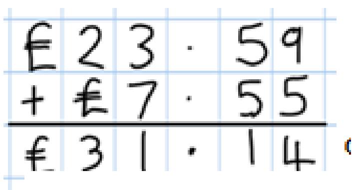
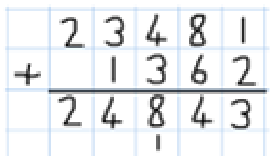
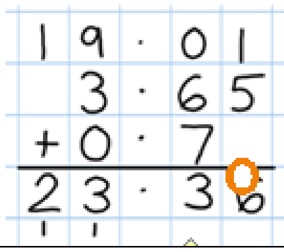
A computer costs 5 times as much as a television. The television costs £429.

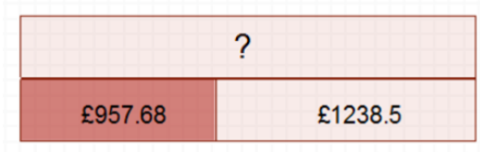
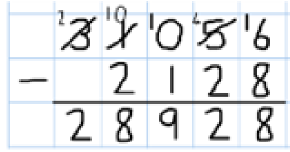
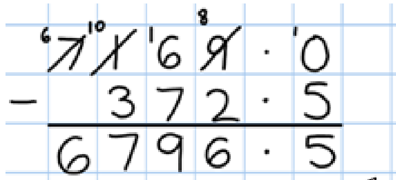
How much does the computer cost?

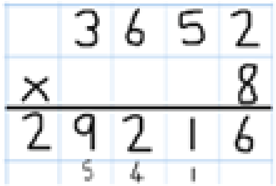
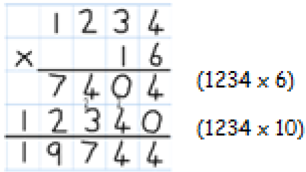
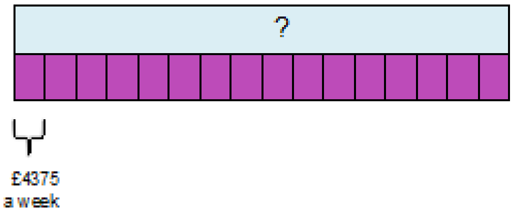
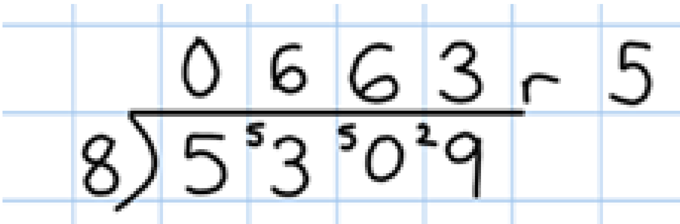
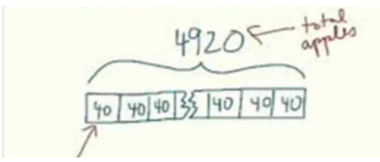


Division	
Dividing up to three digit numbers by a one digit number using short division. Only when the children are secure with dividing a two digit number should they move onto a 3 digit number.	 
Dividing using the bar.	Desmond and Melissa collect cards. They have 192 cards in all. Melissa has three times as many cards as Desmond. How many cards does Desmond have? 

<u>Year 5</u>

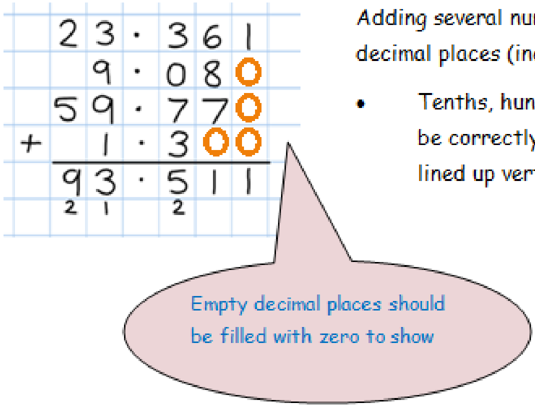
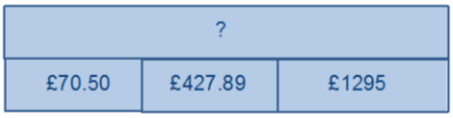
Addition	
Adding numbers with more than 4 digits including decimals *Using place value charts are key to this as well as place value counters to help with the decimals.*	  

Using the bar to find missing digits. *It is important for children to use the bar in this way to encourage the use of it to aid with problem solving.*	This is not a form of getting the correct answer but helping to guide children to the correct operation. MacDonalds sold £9957.68 worth of hamburgers and £1238.5 worth of chicken nuggets. How much money did they take altogether? 
Subtraction	
Subtract with at least four digit numbers including two decimal places. *Include money, measures and decimals ensuring that children do this practically before the abstract.*	Subtract with decimal values, including mixtures of integers and decimals, aligning the decimal point.  Approximate, Calculate, Check . 
Using the bar to find missing digits. *It is important for children to use the bar in this way to encourage the use of it to aid with problem solving.*	A whole to Lapland costs £5005 for a family of four, the Smith's have only saved £3787.75, how much money do they still need to find? 
Multiplication	
Multiplying up to four digit numbers by two digits using long multiplication. *Children need to be taught to approximate first, e.g. for **72 x 38, they will use rounding: 72 x 38** is approximately 70 x 40 = **2800**, and use the approximation to check the*	56 X 27 ______ 392 (56x7) 1120 (56x20) ______ 1512 Approximate, Calculate, Check . · Explain that first we are multiplying the top number by 7 starting with the units. (any carrying needs to be done underneath the numbers).

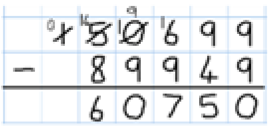
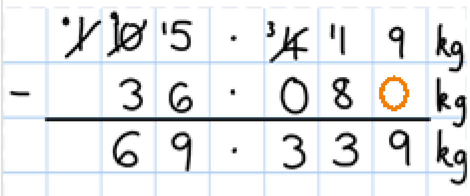
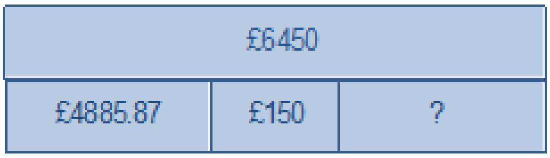
reasonableness of their answer.	· Now explain that we need to put a 0 underneath—explain that this is because we are multiplying the number by 20.. (2 tens) which is the same as multiplying 10 and 2. · Now add the 2 numbers together to give you the answer. · This will need lots of modeling to show the children. 
Using the bar to support multiplication.	The cost to run a sports centre is £4375 a week, how much would it cost to run for 16 weeks? 
Division	
Diving with up to four digit numbers by one digit including numbers where remainders are left.	 **Short division with remainders:** Now that pupils are introduced to examples that give rise to remainder answers, division needs to have a real life problem solving context, where **pupils consider the meaning of the remainder and <u>how</u> to express it,** ie. as a fraction, a decimal, or as a rounded number or value , depending upon the context of the problem.
Using the bar to support division problems.	**<u>Bar Model to support understanding of problem solving:</u>** Frank has 4920 apples. He needs to put them into baskets of 40. How many baskets does he need? 

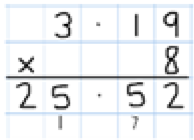
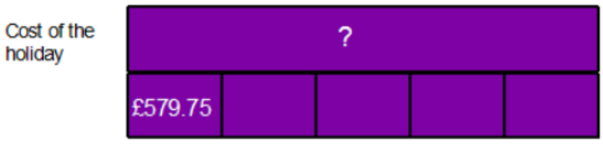
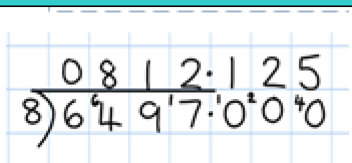
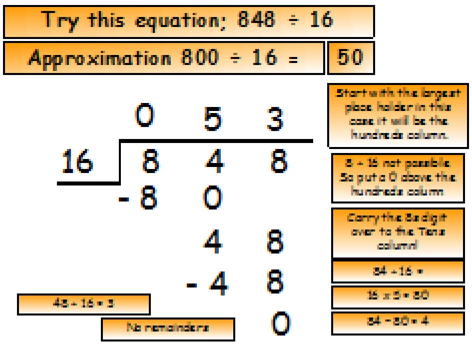
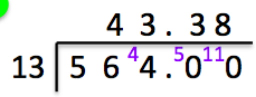
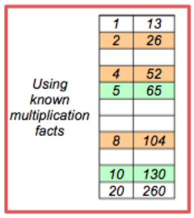
Year 6

Addition

Adding several numbers with up to three decimal places.	 *Adding several numbers with different numbers of decimal places (including money and measures):* • Tenths, hundredths and thousandths should be correctly aligned, with the decimal point lined up vertically including in the answer row.
Adding using the bar.	Jack went on holiday. His flight cost £70.50, the hotel £1295 and spending money £427.89. How much did Jack spend on his holiday? 

Subtraction

Subtracting with increasingly large and more complex numbers and decimal values.	  Very important to use in a range of contexts- measures and money.
Using the bar for subtraction.	Chloe wants to buy a new car for £6450. She has £4885.87 in her savings account. Her Dad gives her £150 for her birthday. How much more money does she need to save? 

Multiplication	
Short and long multiplication with up to two decimal places.	 Approximate, Calculate, Check .
Using the bar to help with multiplication.	If 5 friends went on holiday and each paid £579.75 what was the total cost of the holiday? 
Division	
Divide at least 4 digits by both single-digit and 2-digit numbers (including decimal numbers and quantities)	 **Short division with remainders:** Pupils should continue to use this method, but with numbers to at least 4 digits, and understand how to express remainders as fractions, decimals, whole number remainders, or rounded numbers. Real life problem solving contexts need to be the starting point, where pupils have to consider the most appropriate way to express the remainder.
Long division this is for when dividing by two digit numbers.	 **Division** 564 ÷ 13   564 ÷ 13 $= 43 \text{ r } 5 = 43\frac{5}{13} = 43.4$ (to 1dp)
Using the bar to help divide.	Paul and David hire a car together at a cost of £297.50. Paul pays 6 times more than David. How much does David pay?